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A cohomological approach to the Batalin, Lavrov, Tyutin covariant quantization of irreducible gauge theory

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Abstract. The anti-BRST transformation for an arbitrary irreducible gauge invariant action is implemented in the usual Batalin–Vilkovisky (BV) approach. This is done without duplicating the gauge symmetries, but rather by duplicating all fields and antifields of the theory. In this way the $Sp(2)$-covariant quantization can be accomplished in the standard BV approach and its equivalence with the Batalin–Lavrov–Tyutin (BLT) approach is proved by a special gauge-fixing procedure.

1. Introduction

Without any doubt the most popular and poweful method for the covariant quantization of the gauge field systems is the BRST approach [1–5] (see [6] for a good review of the most important results in this field), both in the Hamiltonian and Lagrangian formalisms. It encompasses the Faddeev–Popov quantization and BRST symmetry, discovered in the context of gauge theories. However, in all these formulations the so-called *non-minimal sector* of the BRST transformations, which is crucial for all applications, is very difficult to understand and to be fixed. In order to overcome these conceptual problems various authors [7–25] have recently tried to employ the anti-BRST ($Sp(2)$) symmetry, thereby identifying the non-minimal sector with part of the minimal one in a natural way.

In the Hamiltonian formalism [7, 9, 13] the anti-BRST symmetry has been introduced by duplication of each first class constraint in an extended phase space. On the other hand, the Lagrangian anti-BRST formalism looks quite different when it is compared with the usual BV formulation [1, 2]. Not only the field content and form of the master equation but also the gauge-fixing process itself is very different, and it seems to be very difficult to see the equivalence of both methods.

In this paper we show that it is possible to reformulate the anti-BRST formalism in the usual BV framework just by *duplication* of all the fields and antifields of the theory and by using homological perturbation theory (HPT) [4, 6]. The crucial point for HPT is the construction of the Koszul–Tate differential δ_K and to generate its *acyclicity*. In fact, the acyclicity of δ_K, which acts in the space of antifields, has been used in [4, 5] (see also [26]) to prove the existence and uniqueness of the solution of the master equation. This property of δ_K determines the spectrum of the antifields and therefore the spectrum of the fields, due to the symmetric structure of fields–antifields. In our approach we shall work only

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with field–antifield pair and we shall *reduce* the construction proposed in [10–12, 14] to the *standard* one. In fact we will show that the antifield–antibracket formulation of the anti-BRST transformation is in fact the usual BV quantization if we introduce an extra assumption that *all fields and ghosts occur in pairs and enter into the quantum action as a sum.*

The basic ingredient of our construction is the observation that if one duplicates the antifields they form a redundant basis of vectors [3], and in order to identify the algebra of polynomials in fields and antifields with the algebra of multivectors [28] it is necessary to set $\Phi^*_{A1} = \Phi^*_{A2}$ in the former algebra,where Φ^*_{Aa} denotes the set of antifields of the theory. Of course this task can be achieved by hand, but the most natural way is to suppose that the BRST differential does this job. This assumption is quite strong and implies precisely the occurrence of the fields in the quantum action only as the sum.

Let us remark that a proposal for the duplication of fields has already appeared in the literature [17–22]. However, we believe that our approach is only superficially similar, since it tries to establish a close connection with the original approach proposed by Batalin and Vilkovisky [1, 2]. Our paper is organized as follows. In section 2 we shall give a short review of the main points of the standard BV theory with emphasis on its HPT aspects. Then in section 3 we treat the $Sp(2)$-symmetric theory as a BV reducible theory. Section 4 is devoted to the gauge-fixing process and to a comparison with the previous theories, and in section 5 we shall present some ideas on the reducible theory.

2. The standard BV theory

In order to set the scene, let us recall the key ingredients of the standard BRST theory in the antibracket–antifield version or BV version. We will adopt a notation so as to establish more explicitly the parallel with the Hamiltonian approach developed in [13, 14, 24].

- We consider a classical system whose dynamics will be governed by an action $S_0(\Phi^i)$ where the field Φ^j has Grassmann parity $\epsilon(\Phi^j) = \epsilon^j$. We denote the equations of motion derived from S_0 by

$$G_j = \frac{S_0 \overleftarrow{\partial}}{\partial \Phi^j}(\Phi) \tag{2.1}$$

where $\overleftarrow{\partial} / \partial \Phi^j$ is the right derivative. Equation (2.1) defines a hypersurface Σ in the manifold of all fields Γ called the *stationary surface* [5].

- The action S_0 could posses gauge invariance and in this case there exist non-trivial relations among the G_i:

$$G_i \cdot R^i_\alpha(\Phi) = 0. \tag{2.2}$$

The set of gauge generators R^i_α with Grassman parity $\epsilon(R^i_\alpha) = \epsilon_i + \epsilon_\alpha$ *is assumed* to be independent and to form a complete set. From the completeness assumption one deduces that the vector fields $R^j_\alpha \cdot \overleftarrow{\partial} / \partial \Phi^j$ verify the equation

$$[R_\alpha, R_\beta] \cong -R_\gamma C^\gamma_{\alpha\beta} \tag{2.3}$$

where $[,]$ is the Lie bracket and $\cong$ means that the equality holds on the stationary surface Σ.

- If gauge invariance is reducible then there exist some functions $Z^\alpha_{\alpha'}$ with $\epsilon(Z^\alpha_{\alpha'}) = \epsilon_\alpha + \epsilon_{\alpha'}$ such that

$$R^j_\alpha \cdot Z^\alpha_{\alpha'} \cong 0. \tag{2.4}$$

The rank of reductibility may be greater than two, in which case one finds relations among the $Z^{\alpha}_{\alpha'}$, etc. We want to emphasize here that in spite of the fact that we have assumed to work with an irreducible theory, the reducibility will occur in the next section due to the duplication of fields and antifields.

- In what follows we shall generically denote by $\{\Phi^A\}$ the collection of all fields which will include the ghosts for all gauge symmetries and possible auxiliary fields arising from the gauge fixing. Then we shall introduce the antifields Φ^*_A which play the role of canonical conjugate variables with respect to a Poisson-like structure defined by the antibracket

$$(F, G) = F \overleftarrow{\partial}_A \cdot \overrightarrow{\partial}^A G - F \overleftarrow{\partial}^A \cdot \overrightarrow{\partial}_A G \tag{2.5}$$

where $\partial_A = \partial/\partial\Phi^A$ and $\partial^A = \partial/\partial\Phi^*_A$ and $\overleftarrow{\partial}$, $\overrightarrow{\partial}$ denote right- and left-derivatives, respectively.
The basic object of any BRST Lagrangian theory is the extended action $S(\Phi^A, \Phi^*_A)$ of ghost number zero, which satisfies *the master equation*

$$(S, S) = 0. \tag{2.6}$$

In addition to (2.6) we have to impose boundary conditions on S in order to obtain a proper solution

$$\begin{aligned} PS &= S_0(\Phi^j) \\ P\, \overrightarrow{\partial}^{\alpha_{k-1}} \cdot S \cdot \overleftarrow{\partial}_{\alpha_k} &= Z^{\alpha_{k-1}}_{\alpha_k}(\Phi^j) \end{aligned} \tag{2.7}$$

where P denotes the projection from the extended field space $\{\Phi^A\}$ to the space of the classical fields $\{\Phi^j\}$ (see [26]).

- The extended action generates, through the antibracket, the BRST symmetry

$$sF = (F, S) \tag{2.8}$$

for any $F = F(\Phi, \Phi^*)$. The linear operator s is a differential and it is an extension of the *Koszul–Tate* (KT) differential $\delta_{\rm K}$ defined by

$$\begin{aligned} \delta_{\rm K}\Phi^*_j &= -G_j \\ \delta_{\rm K}\Phi^*_\alpha &= \Phi^*_j \cdot R^j_\alpha \\ \delta_{\rm K}\Phi^*_{\alpha_1} &= \Phi^*_\alpha \cdot Z^\alpha_{\alpha'} + M_{\alpha'}(\Phi; \Phi^*_j) \end{aligned} \tag{2.9}$$

where $M_{\alpha'}$ is determined by the nilpotence of $\delta_{\rm K}$ and

$$s = \delta_{\rm K} + \cdots . \tag{2.10}$$

The most important property of the KT differential $\delta_{\rm K}$ is its *acyclity*. This means that its cohomology will consist only of the functions on the stationary surface Σ. In the usual BV theory the acyclycity of $\delta_{\rm K}$ has been proved [4, 5, 26], but it could be imposed from the very beginning, and in this case it can determine the antifield structure of the theory. In the last case we must introduce as many antifields as we need for killing *all* the cycle which could appear in the theory.

- The final step in BV method is the construction of a BRST invariant gauge-fixed action. This step is performed by introducing a gauge fermion function $\Psi(\Phi^A)$ that can be chosen to depend only on the fields Φ^A

$$S_\Psi = S\left(\Phi^A, \Phi^*_A = \frac{\overleftarrow{\partial}}{\partial\Phi^A}\Psi\right). \tag{2.11}$$

For a more general setting of gauge-fixing see [16].

3. The $Sp(2)$-symmetric theory as a reducible theory

The basic ideas of embedding an anti-BRST symmetry in the usual BV-approach is achieved by splitting the (total) BRST differential into the sum of two differentials

$$s = s_1 + s_2 \tag{3.1}$$

s_1 being BRST differential and s_2 anti-BRST differential. Due to the fact that s is a differential we have

$$s_1^2 = s_2^2 = s_1 \cdot s_2 + s_2 \cdot s_1 = 0. \tag{3.2}$$

The decomposition (3.1) and equations (3.2) define the BRST–anti-BRST algebra. From the general discussion in section 2 we expect that the Koszul–Tate differentials respectively associated with s_1 and s_2, which are both resolutions of $C^\infty(\Sigma)$, to be anticommuting. This task has been accomplished in the Hamiltonian formalism [13, 14] by *duplicating* the constraints of the theory $G_a(q, p) = 0$, or more precisely by simply repeating them a second time. The corresponding description of the constrained surface is no longer irreducible and we have to use the general theory of reducible constrained systems [4]. However, even in this case we still have a canonical formulation of the theory in which all variables occur in pairs.

On the other hand, in the Lagrangian formalism, in its initial formulation by Batalin, Lavrov and Tyutin [10–12] the symmetry of the pairing of fields with antifields is destroyed, and it is therefore hard to imagine how one could introduce a natural symplectic structure in this case. In this paper we shall try to recover the field $\longleftrightarrow$ antifield symmetry and we shall show that the $Sp(2)$-extended BRST theory is in fact a reformulation of the BV theory in the spirit of duplication of the constraints. As a matter of fact, in the framework of HPT we can consider the equations of motion (2.1) as some constraints which allows us to apply the standard technique [4, 5]. Thus the extended $Sp(2)$ BRST symmetry amounts to a duplication of the equations of motion

$$G_j \rightarrow (G_j, G_j). \tag{3.3}$$

But this duplication can be accomplished by simply duplicating the fields $\Phi^j \rightarrow (\Phi^{j1}, \Phi^{j2})$. In order to have a simple duplication of the equations of motion G_j we must suppose that S_0 has a special dependence on $\Phi^{ja} (a = 1, 2)$, namely

$$S_0 = S_0(\Phi^{j1} + \Phi^{j2}). \tag{3.4}$$

Starting BV quantization with this action , we have to introduce two antifields $\Phi^*_{ja} (a = 1, 2)$. But in this case the action S_0 has an additional gauge symmetry, $\Phi^{ja} \rightarrow \Phi^{ja} + (-1)^a \epsilon^j$, which has nothing to do with the initial gauge symmetry. This additional gauge symmetry implies the existence of new antifields, which will ensure the acyclicity of δ_{K}. These new antifields will be denoted by $\{\overline{\Phi}^j\}$. The Koszul–Tate differential (2.9) should be modified by

$$\delta_{\mathrm{K}} \overline{\Phi}_j = \Phi^*_{j2} - \Phi^*_{j1}. \tag{3.5}$$

The remarkable point here is the fact that (3.5) implies the equality of Φ^*_{j1} and Φ^*_{j2} in the Koszul cohomology. We will suppose that this sort of equality occurs not only for the antifields Φ^*_{ja} with respect to the Koszul cohomology, but also for all antifields with respect to BRST cohomology. In other words we shall suppose that all antifields and fields occur in pairs

$$\Phi^{Aa}, \Phi^*_{Aa} \qquad (a = 1, 2)$$

where we have used the index A as a common index for all fields and ghosts which could occur in the theory. The theory must be developed in such a way that the antifields in any pair $\{\Phi^*_{A1}, \Phi^*_{A2}\}$ coincide in the BRST cohomology. But in this case the quantum action S_T must have the form

$$S_T = \overline{\overline{\Phi}}^A \cdot \left(\Phi^*_{A2} - \Phi^*_{A1}\right) + S(\Phi^a, \Phi^*_a, \overline{\Phi}) \tag{3.6}$$

where $\overline{\overline{\Phi}}^A$ are the fields corresponding to the antifields $\overline{\Phi}_A$ the latter being the generators of the desired BRST cohomology. This fact can be seen easily, if one writes down the BRST transformation of $\overline{\Phi}_A$

$$s\overline{\Phi}_A = -\frac{\overleftarrow{\partial}\, S_T}{\partial\overline{\overline{\Phi}}^A} = \Phi^*_{A1} - \Phi^*_{A2}. \tag{3.7}$$

But this form of S_T is quite restrictive, since it has in addition to obey the master equation. Thus, by virtue of (3.5), the master equation $(S_T, S_T) = 0$ is equivalent to the set of equations

$$\frac{S\,\overleftarrow{\partial}}{\partial\Phi^{A1}} - \frac{S\,\overleftarrow{\partial}}{\partial\Phi^{A2}} = 0 \tag{3.8}$$

and

$$\tfrac{1}{2}(S, S) + \left(\Phi^*_{A1} - \Phi^*_{A2}\right) \cdot \frac{\partial S}{\partial\overline{\Phi}_A} = 0. \tag{3.9}$$

Equations (3.9) imply that S must have a special dependence on the fields $\left\{\Phi^{Aa}\right\}$:

$$S = S\left(\Phi^{A1} + \Phi^{A2}, \Phi^*_{Aa}, \overline{\Phi}_A\right). \tag{3.10}$$

The BRST transformation generated by S_T takes the form

$$sF = (F, S) + VF \tag{3.11}$$

where $F = F(\Phi, \Phi^*, \overline{\Phi})$ and V is a nilpotent operator introduced by Henneaux [3] which has the form

$$V = \left(\Phi^*_{A2} - \Phi^*_{A1}\right) \frac{\overrightarrow{\partial}}{\partial\overline{\Phi}_A}.$$

The master equation (2.6) can now be solved in the usual way by expanding S with respect to the grading of the Koszul–Tate differential [5], i.e. with respect to the antighost number. However, as we have already pointed out, this differential must be acyclic and its acyclicity determines the antifield content of the theory.

With the general results (3.6) and (3.10) we can now start to construct the Koszul–Tate differential δ_K for an irreducible theory and to determine the antifield spectrum. We want to emphasize again that simply demanding that the differential δ_K be acyclic forces the antifield spectrum to be just the correct one [5]. First of all, the first equation from (2.9) now becomes

$$\delta_K\Phi^*_{ja} = -G_{ja} = -S_{0,ja} \qquad a = 1, 2 \tag{3.12}$$

and the equality of these two equations implies (3.4). On the other hand, if S_0 posseses a gauge invariance, one finds some additional non-trivial cycles. Because the functions R^j_α are all independent a basis of these non-trivial cycles must contain the terms

$$\Phi^*_{j2} - \Phi^*_{j1} \qquad \Phi^*_{ja} \cdot R^j_\alpha. \tag{3.13}$$

Therefore, in order to recover the acyclicity of $\delta_{\rm K}$ we have to add (following Tate) additional variables, which are just extra antifields in the same way as in a reducible theory [5]. The antifields $\overline{\Phi}_j$ introduced to kill the first cycle from (3.13) play a special role and they are called 'bar' fields (or antifields). They have already been introduced in (3.5) and therefore appear in the general form of the solution of the master equation (3.6). The antifields $c^*_{\alpha a|b}$ are introduced to kill the cocycles $\Phi^*_{ja} \cdot R^j_\alpha$, i.e.

$$\delta_{\rm K} c^*_{\alpha b|a} = \Phi^*_{ja} \cdot R^j_\alpha. \tag{3.14}$$

One clearly has $\delta^2_{\rm K} = 0$ and we have added an extra $Sp(2)$ index to $c^*_{\alpha a|b}$ since we take into account the fact that all antifields must occur in pairs and both their BRST and Koszul–Tate (given by $\delta_{\rm K}$) transformations must be equal. But in this case, in spite of the fact that we started from irreducible theory, the procedure must be continued and we have to add three *independent* additional terms to the basis of non-trivial cycles (3.13), which may be chosen as

$$c^*_{\alpha 2|a} - c^*_{\alpha 1|a} \qquad c^*_{\alpha 2|1} - c^*_{\alpha 1|2} + \overline{\Phi}_j \cdot R^j_\alpha.$$

In order to recover the acyclicity of $\delta_{\rm K}$ again, one must introduce further antifields $\overline{c}_{\alpha a}$ and $B^*_{\alpha a}$ and define

$$\begin{aligned} \delta_{\rm K}\overline{c}_{\alpha a} &= c^*_{\alpha 2|a} - c^*_{\alpha 2|a} \\ \delta_{\rm K} B^*_{\alpha a} &= c^*_{\alpha 2|1} - c^*_{\alpha 1|2} + \overline{\Phi}_j \cdot R^j_\alpha. \end{aligned} \tag{3.15}$$

For the irreducible theory we need only one extra 'bar' antifield $\overline{B}_\alpha$ such that

$$\delta_{\rm K}\overline{B}_\alpha = B^*_{\alpha 2} - B^*_{\alpha 1}. \tag{3.16}$$

The demand that the differential $\delta_{\rm K}$ be acyclic automatically forces the antifield spectrum to be just a duplication of the correct set of minimal and non-minimal sectors of the standard BV theory [1,2].

In conclusion we can say that the process of defining the Koszul–Tate differential consists of two steps: in the first step we have killed the cocycles which contain only old antifields and in the second step we have killed the cocycles containing new antifields, introduced in the first step. This process is different from the one used in the standard BV method described in section 2.

Having settled the complete form of the Koszul–Tate differential we can now see whether it allows a decomposition $\delta_{\rm K} = \delta_1 + \delta_2$ necessary for any $Sp(2)$ theory. This can be verified simply by inspection if one defines the following differentials:

$$\begin{aligned} \delta_a \Phi^*_{ib} &= -S_{0,ia}\delta_{ab} \text{ (no sum)} \\ \delta_a c^*_{\alpha b|c} &= (\Phi^*_{jc} R^j_\alpha)\delta_{ab} \\ \delta_a B^*_{\alpha b} &= (-\epsilon^{cd} c^*_{\alpha,c|d} + \overline{\Phi}_j R^j_\alpha)\delta_{ab} \end{aligned} \tag{3.17}$$

and

$$\delta_a \overline{\Phi}_A = (-1)^a \Phi^*_A \tag{3.18}$$

where $A = (j, \alpha b, \alpha)$ and ϵ^{ab} is the invariant tensor of the $Sp(2)$ group with $\epsilon^{12} = -\epsilon^{21} = 1$ and $\epsilon^{11} = \epsilon^{22} = 0$. It is easy to check that the quantum action S must start as follows:

$$S = S_0 + \Phi^*_{ja} R^j_\alpha (c^{\alpha 1|a} + c^{\alpha 2|a}) - (\epsilon^{ab} c^*_{\alpha a|b} - \overline{\Phi}_j R^j_\alpha)(B^{\alpha 1} + B^{\alpha 2}) + \cdots . \tag{3.19}$$

Hitherto we have not tried to make any distinction between the antifields Φ^*_{A1} and Φ^*_{A2}. But this distinction is crucial in any $Sp(2)$-covariant quantization. Therefore we shall ascribe the following values of the ghost number to the antifields:

$$\left.\begin{array}{l} gh(\Phi^*_{Aa}) = (-1)^a - gh(\Phi^A) \\ gh(\overline{\Phi}_A) = -gh(\Phi^A) \end{array}\right\} \qquad a = 1, 2. \tag{3.20}$$

Now if we apply the general formalism of HPT, developed for the usual theory in [5] or [4] one can conclude that (3.9) not only has a solution but it can also be taken to be of bidegree (0, 0) in the conventions introduced by Grégoire and Henneaux [13, 15].

Taking into account the ghost numbers of the fields and antifields, any antibracket can be split into two parts

$$(A, B) = (A, B)^1 + (A, B)^2 \tag{3.21}$$

where $(A, B)^a$ $(a = 1, 2)$ is the antibracket (2.5) built with the pair (Φ^{Aa}, Φ^*_{Aa}). They have different ghost numbers and the master equation splits into two equations

$$\tfrac{1}{2}(S, S)^a + V^a S = 0 \tag{3.22}$$

where

$$V^a = \epsilon^{ab}\Phi^*_{Ab}\frac{\partial}{\partial\overline{\Phi}_A}. \tag{3.23}$$

Moreover the BRST differential s splits as in (3.1) with

$$s_a = \tfrac{1}{2}(.,S)^a + V^a \qquad a = 1, 2. \tag{3.24}$$

For the irreducible theories with a closed algebra, we can find an exact and rather simple solution of the master equation. Such theories are characterized by the fact that the algebra of their generators (2.3) closes off the stationary surface Σ and the solution of any equation of the form $R^j_\alpha X^\alpha = 0$ is $X^\alpha = 0$. This is not a big restriction since the majority of the theories relevant to practical application belong to this class (e.g. gravity, supergravity, Yang–Mills theory, Chern–Simons theory, etc). From the point of view of the *usual* BRST quantization of all these theories, the typical fact is that the master equation with the standard boundary conditions

$$S = S_0 + \Phi^*_j \cdot R^j_\alpha \cdot c^\alpha + \cdots \tag{3.25}$$

has a very simple solution

$$\begin{aligned} S &= S_0 + \Phi^*_j \cdot R^j_\alpha \cdot c^\alpha + \tfrac{1}{2}c^*_\alpha C^\alpha_{\beta,\gamma}c^\gamma c^\beta + \overline{c}^*_\alpha \cdot B^\alpha \\ &= S_0 + \Phi^*_A \cdot (s\Phi^A) \end{aligned} \tag{3.26}$$

where the field–antifield structure of the theory is given by

- The initial fields $\{\Phi^j\}$ and the corresponding antifields $\{\Phi^*_j\}$ with $gh(\Phi^j) = 0$ and $gh(\Phi^*_j) = 1$ with the BRST symmetry

$$s\Phi^j = R^j_\alpha \cdot c^\alpha. \tag{3.27}$$

- The ghosts and antighosts $\{c^\alpha, c^*_\alpha\}$ with $gh(c^\alpha) = 1$ and $gh(c^*_\alpha) = -2$ with the BRST symmetry

$$sc^\alpha = \tfrac{1}{2}C^\alpha_{\beta,\gamma}c^\beta c^\gamma. \tag{3.28}$$

- The non-minimal sector with the ghost fields and antifields conjugate with c^α, $\{\overline{c}^\alpha, \overline{c}^*_\alpha\}$ and the Nakanishi–Lautrup fields and antifields $\{B^\alpha, B^*_\alpha\}$ with the BRST symmetry

$$s\overline{c}^\alpha = B^\alpha \qquad sB^\alpha = 0. \tag{3.29}$$

From the point of view of the extended BRST quantization the field–antifield structure of these theory and the boundary conditions are different.

- Instead of the initial fields $\{\Phi^i\}$ we have the pair of fields $\{\Phi^{ja},\ a=1,2\}$ and their antifields $\{\Phi^*_{ja}\}$.
- The ghosts and their conjugate occur now together as $\{c^{\alpha a|b}\}$ with the corresponding antifields $\{c^*_{\alpha a|b}\}$.
- The Nakanishi–Lautrup fields occur in pairs $\{B^{\alpha a}\}$ and correspond to the antifields $\{B^*_{\alpha a}\}$.
- *In addition, in the extended theory we must introduce the 'bar' fields* $\{\bar{\Phi}_j, \bar{c}_{\alpha a}, \bar{B}_\alpha\}$.

The master equation (2.6) with the boundary conditions (3.19) yield

$$S = S_0 + \Phi^*_{Aa} s^a \tilde{\Phi}^A + \tfrac{1}{2}\epsilon^{ab}\bar{\Phi}_A s_a s_b \tilde{\Phi}^A \tag{3.30}$$

where

$$\begin{aligned} s^a \Phi^{jb} &= R^j_\alpha \cdot \tilde{c}^{\alpha a|b} \\ s^a B^{\alpha b} &= -\tfrac{1}{2} C^\alpha_{\beta\gamma} \tilde{B}^\beta \tilde{c}^{\gamma a} - \tfrac{1}{12}(-1)^{\epsilon_\beta}(2C^\alpha_{\gamma\beta,j} R^j_\rho + C^\alpha_{\gamma\sigma} C^\sigma_{\beta\rho})\tilde{c}^{\rho b}\tilde{c}^{\beta a}\tilde{c}^{\gamma c}\epsilon_{cd} \\ s^a c^{\alpha b|c} &= -\epsilon^{ab}\tilde{B}^\alpha - \tfrac{1}{2} C^\alpha_{\beta\gamma}\tilde{c}^{\gamma b}\tilde{c}^{\beta a}. \end{aligned} \tag{3.31}$$

and we have used the tilde (˜) as an abbreviation for all the sum of fields pair

$$\tilde{\Phi}^A = \Phi^{A1} + \Phi^{A2}.$$

A direct verification shows that (3.30) with (3.31) is indeed a solution of the master equation (2.6). In the solution of the master equation we have used the Jacobi identity

$$(-1)^{\epsilon_\beta \epsilon_\rho}(C^\alpha_{\beta\gamma,j} R^j_\rho + C^\alpha_{\beta\sigma} C^\sigma_{\gamma\rho}) + \text{cycle}(\beta,\gamma,\rho) = 0.$$

4. The gauge-fixing procedure

In the standard BV theory the gauge-fixed action is obtained by the simple replacement

$$\Phi^*_A = \frac{\partial\Psi}{\partial\Phi^A}$$

where Ψ is a fermionic gauge-fixing function which, at least in principle, is arbitrary. However in the $Sp(2)$ theory it cannot be chosen arbitrarily but must have a special form. Its form has been discussed by Batalin, Lavrov and Tyutin [10, 11]. Unfortunately their approach, which is absolutely correct, is not very standard and it is difficult to see the equivalence with the usual BV procedure. However, Grégoire and Henneaux [14, 15] have shown that the BLT gauge-fixing prescription can be understood in a somewhat modified standard framework. However, they were forced to introduce some auxiliary fields, which could not entirely be justified in their framework. We will show that the BLT procedure is just *the standard BV procedure* accommodated to our formulation of BRST symmetry.

We will try to use the gauge-fixing procedure to get rid of one of the fields from any pairs. This can be done if we employ the freedom in the definition of fields and antifields up to canonical transformations (see [26, 6]). Using this freedom we may *regard* Φ^*_{A2} as *fields* and Φ^{A2} as *antifields* in what follows.

The total quantized action S_T can be written as

$$\begin{aligned} S_T &= S_T(\Phi^{A1} + \Phi^{A2}; \Phi^*_{A1}, \Phi^*_{A2}; \overline{\overline{\Phi}}_A, \overline{\Phi}_A) \\ &= S(\Phi^{A1} + \Phi^{A2}; \Phi^*_{A1}, \Phi^*_{A2}, \bar{\Phi}) + \overline{\overline{\Phi}}^A \cdot (\Phi^*_{A1} - \Phi^*_{A2}). \end{aligned} \tag{4.1}$$

Taking into account canonical transformations, the gauge-fixed action becomes

$$S_T = S\left(\Phi_1 + \frac{\partial\Psi}{\partial\Phi_2^*}, \Phi_1^* = \frac{\partial\Psi}{\partial\Phi_1}, \Phi_2^*, \frac{\partial\Psi}{\partial\overline{\overline{\Phi}}}\right) + \overline{\overline{\Phi}} \cdot \left(\frac{\partial\Psi}{\partial\Phi_1} - \Phi_2^*\right). \tag{4.2}$$

Let us chose the gauge fermion Ψ as

$$\Psi = \overline{\overline{\Phi}}^A \cdot \frac{\partial F}{\partial\Phi^{A1}} \tag{4.3}$$

where F is a bosonic function which depends only on Φ^{A1}. For this choice of the fermion function the gauge-fixed action becomes

$$S_\Psi = S\left(\Phi^A; \overline{\overline{\Phi}}^A \cdot \frac{\overleftarrow{\partial}^2 F}{\partial\Phi^A\partial\Phi^B}, \Phi_{A2}^*, \frac{\partial F}{\partial\Phi^A}\right) - \overline{\overline{\Phi}}^A \cdot \Phi_{A2}^* \tag{4.4}$$

where we have used the notation $\Phi = \Phi_1$. In the last equation we have been used the fact that the term

$$\overline{\overline{\Phi}}^A \cdot F_{AB} \cdot \overline{\overline{\Phi}}^B = 0$$

where $F_{AB} = \overleftarrow{\partial}^2 F/\partial\Phi^A \cdot \partial\Phi^B$, since the fields $\overline{\overline{\Phi}}^A$ and Φ^A have opposite Grassmann parities.

The expression for the gauge-fixed action can be somewhat simplified and transformed into the form given by Batalin, Lavrov and Tyutin if we introduce Lagrange multipliers for the gauge choice

$$\bar{\Phi}_A = \frac{\overleftarrow{\partial} F}{\partial\Phi^A} \qquad \Phi_{A1}^* = \overline{\overline{\Phi}}^B \frac{\overleftarrow{\partial}^2 F}{\partial\Phi^A\partial\Phi^B}. \tag{4.5}$$

At the level of path integration

$$S_{\text{eff}} = S(\Phi^A; \Phi_{A1}^*, \Phi_{A2}^*; \bar{\Phi}^A) - \overline{\overline{\Phi}}^A \cdot \Phi_{A2}^* + \pi^A\left(-\Phi_{A1}^* + \frac{\overleftarrow{\partial}^2 F}{\partial\Phi^A\partial\Phi^B}\right) \cdot \overline{\overline{\Phi}}^B$$
$$+ \left(\bar{\Phi}_A - \frac{\overleftarrow{\partial} F}{\partial\Phi^A}\right) \cdot \lambda^A \tag{4.6}$$

becomes equivalent to S_Ψ by integrating out the Lagrange multipliers π^A and λ^A.

The action (4.6) is exactly the one used in the path integral approach proposed by Batalin, Lavrov and Tyutin [10]. Nevertheless, this action has been obtained here within the standard BV formalism, and so the equivalence between the extended BRST and usual BV formalisms is manifest. Therefore we can apply the standard Batalin–Vilkovisky theorem [1, 2] and conclude that the path integral is independent of the form of the bosonic functional F. Here we want to emphasize that in our gauge-fixing procedure we have not introduce any new extra fields (see [14]), since they had been introduced in a natural way in the construction of the quantum action.

5. The reducible case

The general case of the reducible theory could be developed along the same lines, but the Koszul–Tate differential and the structure of the antifield spectra in this case is much more involved [29]. This spectrum is again obtained by killing all possible non-trivial cocycles.

The details will be given elsewhere, but here we want to give a simple example of how to construct the Koszul–Tate differential. Let us suppose that we want to quantize a reducible theory with one stage of reducibility. In this case the generators of the gauge transformations R^j_α are not independent and they satisfy the relations

$$Z^\alpha_{\alpha_1} \cdot R^j_\alpha = 0. \tag{5.1}$$

In the first step we have to kill the following non-trivial cocycles:

$$\begin{array}{lll} \Phi^*_{j2} - \Phi^*_{j2} & \Phi^*_{ja} R^j_\alpha & \\ c^*_{\alpha 2|a} - c^*_{\alpha 1|a} & c^*_{\alpha 2|1} - c^*_{\alpha 1|2} + \bar{\Phi}_j R^j_\alpha & Z^\alpha_{\alpha_1} c_{\alpha a|b} \end{array} \tag{5.2}$$

and we have to introduce the new antifields $\{\bar{\Phi}^*_j, \bar{c}_{\alpha a}, B^*_{\alpha a}, \bar{B}_\alpha$ and $c^*_{\alpha_1 a|bc}\}$.

In the second step we have to kill the non-trivial cocycles

$$\begin{array}{l} c^*_{2|ab} - c^*_{1|ab} \\ c^*_{\alpha_1 2|1a} - c^*_{\alpha_1 1|2a} + (\bar{c}_{\alpha a} - \frac{1}{2} B^*_{\alpha a}) Z^\alpha_{\alpha_1} \end{array} \tag{5.3}$$

and therefore we have to introduce the new antifields $\bar{c}_{\alpha_1 ab}$ and $B^*_{\alpha_1 a|b}$. Finally we obtain the last non-trivial cocycles

$$B^*_{\alpha_1 2|a} - B^*_{\alpha_1 1|a} \tag{5.4}$$

and in order to kill these we introduce the 'bar' fields $\bar{B}_{\alpha_1 a}$.

We conclude this short section with a presentation of the set of antifields which are used in an one-stage reducible theory in the extended BRST quantization. There are two sets of *antifields:*

- The 'star' antifields

$$\Phi^*_{Aa} = (\phi^*_{ja}; c^*_{\alpha a|b}, c^*_{\alpha_1 a|bc}; B^*_{\alpha a}, B^*_{\alpha_1 a|b}).$$

- The 'bar' fields

$$\bar{\Phi}_A = (\bar{\Phi}_j; \bar{c}_{\alpha|a}; \bar{c}_{\alpha_1|ab}).$$

This structure is the same as that proposed in [10, 11] just by using the $Sp(2)$ invariance of the theory. Thus we can conclude that the acyclicity of Koszul–Tate differential and the hypothesis that all fields occur in the quantum action as in (3.6) lead us to an antifield spectrum in agreement with that introduced in [10].

The detailed structure of an arbitrary reducible theory and the form of the solution of the master equation in this case will be given elsewhere [29].

6. Conclusion

We have shown that the $Sp(2)$ version of the BRST-symmetry, proposed by Batalin, Lavrov and Tyutin [9, 10, 11, 14], can be accomplished in the standard BV antifield–antibracket formalism [1, 2, 6] by adopting the Koszul–Tate cohomology for the case where all fields are duplicated. We have also shown that the solution of the standard master equation, with the boundary conditions given by the Koszul–Tate cohomology, coincide with that given in [10, 7, 14]. Furthermore, the gauge-fixing process can be achieved by a particular choice of the gauge-fixing fermion in the standard BV formalism.

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